OPTICS AND THE LINE IN PLATO'S *REPUBLIC*

SOCRATES, in the *Republic* (509 d–511 e), uses the symbol of a divided line to illustrate the distinction between the Visible and Intelligible Worlds, and between the kinds of perception appropriate to each.[1] This paper will present a new hypothesis: that the proportions of the line are derived from optical theory.

The construction of the Divided Line is described as follows: Socrates asks his interlocutors to represent the Visible and Intelligible Worlds by a line divided into two unequal segments. (See Diagram I, below, where line AA' is divided at C.) The ratio in which the division is to be made is not specified, and it seems that any ratio is acceptable provided that one segment is longer than the other. Socrates then tells them to cut each part again according to the same ratio as the original division. (In Diagram I, below, AC is divided at F, and CA' is divided at F'.) After describing the division of the line thus into four parts, Socrates goes on to explain the philosophical significance of each part. For the purposes of this paper the following brief identification of each segment of the line will suffice.[2]

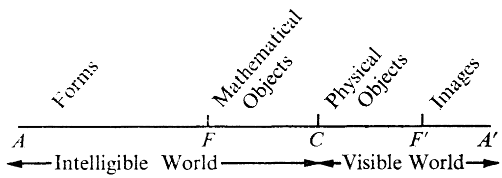

Diagram I

Segment CA', composed of CF' and $F'A'$, represents the Visible World. Subsection $F'A'$ symbolizes things seen indirectly, such as images and reflections:[3] CF' symbolizes physical objects, as they are seen directly. Segment AC, composed of AF and FC, represents the Intelligible World. Subsection FC symbolizes the objects treated by geometry and related mathematical sciences; AF represents the highest class of objects, commonly labelled 'Forms' or 'Ideas'.

The proportions of the Divided Line may be expressed as follows: $\frac{AA'}{AC} = \frac{AC}{CA'}$ and $\frac{AC}{AF} = \frac{AF}{FC}$ and $\frac{CA'}{CF'} = \frac{CF'}{F'A'}$. This ratio is the famous 'Golden Section'

[1] I wish to express my thanks to Professor Martin Ostwald of Swarthmore College for his invaluable suggestions during the writing of this paper.

[2] For detailed discussions on the actual contents of the line see J. L. Stocks, 'The Divided Line', *CQ* v (1911), 72–88; A. S. Ferguson, 'Plato's Simile of Light', *CQ* xv (1921), 131–52, and xvi (1922), 15–28; A. S. Ferguson, 'Plato's Simile of Light Again', *CQ* xxviii (1934), 190–210; J. E. Raven, 'Sun, Divided Line, and Cave', *CQ* N.S. iii (1953), 22–32; J. Ferguson, 'Sun, Line, and Cave Again', *CQ* N.S. xii (1963), 188–93; and R. G. Tanner, *CQ* N.S. xx (1970), .

[3] According to Proclus 289. 21 εἰκών is used elsewhere by Plato with many connotations, but the meaning is limited here to those images which appear by means of the power of light. See J. L. Stocks, loc. cit. 88.

widely used in Ancient Greece.[1] A necessary consequence of division of any line according to the 'Golden Section' is that the two central segments are always equal in length—a fact which Plato could scarcely help but know. Yet, in view of the philosophical doctrine illustrated by the Divided Line, the equality of the central segments has been regarded as a defect, or unintended feature of the line. According to J. Adam, *The Republic of Plato* (Cambridge, 1902), 64: 'This last equality—so far as it goes—is a slight, though unavoidable defect in the line, for [the two central segments are not equal to each other] in point of clearness.' Likewise, D. Ross, *Plato's Theory of Ideas* (Oxford, 1961), 45–6, wrote: 'The inference has sometimes been drawn that Plato deliberately means the two middle sections to be equal, and therefore cannot mean the four subsections to stand for four kinds of object increasing in clearness or reality. But the equality of [the central segments], though it follows from the ratios prescribed, is never mentioned; and on the other hand the passage contains clear indications that the four subsections are meant to stand for four divisions of being of increasing "clearness" (509 d 9) or "truth" (510 a 9). The equality of the middle subsections is an unintended, and perhaps by Plato unnoticed, consequence of what he does wish to emphasize, that the subsections of each section, and the sections themselves, stand for objects unequal in reality.' According to Ross, if the mathematics of the line had allowed $F'C$ (in my Diagram I, above) to be in the same ratio to CF as $A'C$ is to CA, $A'F'$ to $F'C$, and CF to FA, 'he would have had it so; and the fact that this is mathematically impossible is only an indication of the fact that the line, being but a symbol, is inadequate to the whole truth which Plato meant to symbolize'. A series of articles in the *Classical Quarterly* dealing specifically with the Line tends to ignore the fact of the equality of the central segments,[2] and A. Wedberg, *Plato's Philosophy of Mathematics* (Stockholm, 1955), 102–3, dismissed the problem by stating that it 'is obviously an unintended feature of the mathematical symbolism to which no particular significance should be attached'.

Plato, as is well known, was a devoted student of geometry and mathematics, and frequently elucidated his philosophical doctrines with illustrations drawn from these sciences.[3] It seems unlikely that, as a mathematician, after describing the Line in detail he would have been unaware that the central segments were equal in length—when this equality becomes obvious to the casual reader who draws the Line in accordance with the instructions given in *Republic* 509 d.

In the section of the *Republic* (509 a–510 c) where the Line is described a large number of words related to the mechanism of vision—Visible World, images, shadows, reflections—are used.[4] Moreover, optical phenomena form the basis of the philosophical discussion in the chapters immediately preceding and following the description of the Line. Here we may note especially that in *Republic* 508 Socrates makes an analogy between the physical sense of vision, which functions adequately only in the presence of sunlight, and the vision the Soul has when it views objects in the presence of truth and reality. Moreover, the Allegory of the Cave (*Republic* 514 a–518 b), which follows the description of the Divided Line, is replete with references to optical matters such as images, shadows, light, and darkness.

[1] See R. Brumbaugh, *Plato's Mathematical Imagination* (Bloomington, 1954), 279 n. 23.

[2] Above, p. 389 n. 2.

[3] Brumbaugh, op. cit. 3–7.

[4] The stem *ὁρα-* appears six times; *εἰκών* four times; *ὄψις*, *σκιά*, *φανός*, *φαντάσματα*, *φῶς*, *σαφήνεια*, and *ἀσάφεια* each appear once.

Plato's interest in optical phenomena may be observed elsewhere in his work. In *Republic* 602 c, he notes that the apparent size of an object depends upon the distance of the viewer from it, and that a straight stick seems bent when it is seen partially submerged in water.[1] In *Sophist* 266 b, in *Theaetetus* 193 c, and in *Timaeus* 46 a–c, he discusses reflections in plane and curved mirrors.

References to optics in Plato's work are by no means unexpected when we consider that he was part of an intellectual tradition which frequently employed examples from the mathematical sciences in philosophical discourse. Unfortunately, because the extant optical treatises of Greek scientists[2] were all written after Plato's death there is no available evidence concerning Plato's possible knowledge of optical theory. However, among the earliest Greek scientists whose work on optics is in some form extant or known, Euclid (born *c.* 323 B.C.) and Archimedes (born *c.* 287 B.C.) can definitely be said to have studied the properties of mirrors, and Apollonius of Perga (born *c.* 262 B.C.) certainly knew of the focal points of various curved mirrors.[3]

Literary and archaeological evidence indicates that not only mirrors but also magnifying glasses were used in antiquity. In Aristophanes' *Clouds* 768 (423 B.C.) there is mention of a ὕαλος, described as a λίθος διαφανὴς ἀφ' ἧς τὸ πῦρ ἅπτουσι. LSJ, *A Greek–English Lexicon* (Oxford, 1940), 1840, translate ὕαλος in Aristophanes as follows (italics theirs): '*a convex lens of crystal*, used as a burning-glass.' In fact, magnifying glasses dating from the sixth to fourth centuries B.C. have been found.[4] Although extant early Greek optical treatises do not discuss the properties of lenses, these properties may have been known by analogy with the properties of mirrors, or observed empirically, as, for example, Plato observed the reflections in various curved mirrors.

Perhaps a knowledge of optics suggested to Plato the proportions of the Divided Line. At this point it will be illuminating to compare the Divided Line (drawn in Diagram I) with the line which illustrates the lens equation.

In Diagram II a common magnifying glass—a double convex lens—is shown with its centre at C. The focal points, F and F', of the lens are equidistant from C. An object, AB, is viewed through the lens. To determine graphically the location of the image of this object, one ray of light is traced from B to the lens, and is deflected through the focal point F'. Another ray is drawn from B through the centre of the lens, C, and suffers no deflection. The point where the two rays cross, that is B', determines the place where the image, $A'B'$, of the object, AB, appears. The image is inverted relative to the object. We can see that the line AA', in Diagram II, is divided at C. Segment AC is divided at F, and CA' is divided at F'. Without reproducing here the simple

[1] The phenomena observed by Plato exist irrespective of his theory of the mechanics of vision involving the emanation of rays from the observer and the object observed.

[2] For an analysis of early work on optics see Sir Thomas L. Heath, *A History of Greek Mathematics* (Oxford, 1921), ii. 3, 200–3 *et passim*.

[3] See T. L. Heath, 'The fragment of Anthemius on burning mirrors and the "Fragmentum mathematicum Bobiense" ', *Bibliotheca Mathematica*, Ser. 3, vii (1906–7), 225–33; especially 232–3.

[4] For a discussion of ancient magnifying glasses, including nine from the sixth to fourth centuries B.C. which were displayed in the Lavigerie Museum in Carthage, see H. C. Beck, 'Early Magnifying Glasses', *Antiquaries Journal* vii (1928), 327–30. Beck observes that in antiquity magnifying glasses were probably used commonly for such precision work as gem cutting and die sinking. It is likely that such glasses were far more abundant than archaeological evidence would indicate, but, owing to the fragility of glass, they have not survived.

geometric proof of the lens equation (which may be found in any elementary physics textbook), we may observe that the proportions of line AA' in Diagram II are exactly the same as the proportions of line AA' in Diagram I

$$\left(\frac{AC}{CA'}=\frac{AF}{FC}=\frac{CF'}{F'A'}, \quad FC=CF'\right).$$

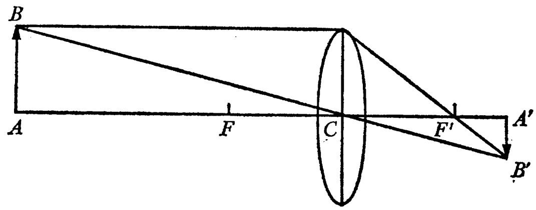

Diagram II

Let us consider the object at AB as a particular Form or Idea, naturally located at the primary point in the segment (AF) which represents the Forms. Likewise the image at $A'B'$ appears at the end of the segment ($F'A'$) which represents images. The equality of the central segments may perhaps be explained by referring to *Republic* 510 d, where students of geometry are described as really thinking of the Abstract Square and Diagonal (of segment FC), although in their inquiries they must employ visible imperfect squares and diagonals (of segment CF'), as if the perfect intelligible Square and the man-made representation of a square were equivalent. But the lens placed at the boundary between the Intelligible World and the Visible World emphasizes the distinction between the two. Since Plato was a mathematician, and presented the Line in a context concerned with optical phenomena, it seems reasonable to hypothesize that he had the properties of a lens in mind when he described the division of the Line.

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